

Hidden Magnetism in Periodically Modulated One Dimensional Dipolar Fermions

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The experimental realization of time dependent ultracold lattice systems has paved the way towards the implementation of new Hubbard-like Hamiltonians. We show that in a one dimensional two components lattice dipolar Fermi gas the competition between long range repulsion and correlated hopping induced by periodically modulated on-site interaction allows for the formation of exotic hidden magnetic phases. The magnetism, characterized solely by string-like nonlocal order parameters, manifests itself both in the charge and, noticeably, in the spin degrees of freedom. Such behavior is enlighten by employing both Luttinger theory and numerical methods. Crucially the range of parameters for which hidden magnetism is present can be reached by means of the currently available experimental setups and probes.

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Since the Haldane's discover in 1983 [1], hidden magnetic orders have attracted huge interest. In this context two very recent experiments involving organic molecular compounds [2] and an oxide of nickel spin chain [3] have obtained relevant results. Nowadays cold atomic systems offer an ideal platform to simulate fundamental quantum physics [4]. Indeed proposals for the realization of hidden charge magnetism [5–9] have been carried out. Whereas the possible realization of hidden spin orders in fermionic systems is still an unexplored scenario.

At the same time investigations of periodically modulated quantum systems [10] have predicted very interesting effects [11–13]. They have stimulated impressive experimental achievements like frustrated classical magnetism [14], gauge potentials [15], ferromagnetic domains [16] and the realization of new particle-hole symmetric Hubbard-like Hamiltonians with correlated hopping processes (CHPs) [17]. The latter are believed to be responsible for fundamental still open questions [18], one of these being the celebrated η -superconductivity [19].

A configuration closer to real materials [26] can be realized in trapped ultracold atomic systems with strong long-range dipolar interaction, like magnetic atoms [20–22] and polar molecules [23–25]. In case of *Er* magnetic atoms, this research line has produced the recent experimental realization [27] of a paradigmatic model in condensed matter, the extended Bose-Hubbard model. Furthermore out-of-equilibrium dipolar systems have been both used to generate quantum magnetic Hamiltonians [28, 29] and proposed to study disorderless many-body localized regimes [30].

Motivated by the aforementioned reasons, in this paper we investigate the properties of a dipolar fermionic mixture subject to a rapid time periodic modulation of the on-site interaction and trapped in a one-dimensional (1D) optical lattice (OL). In this regime Floquet theory can be applied. It allows to derive an effective time independent model where an additional term of CHP

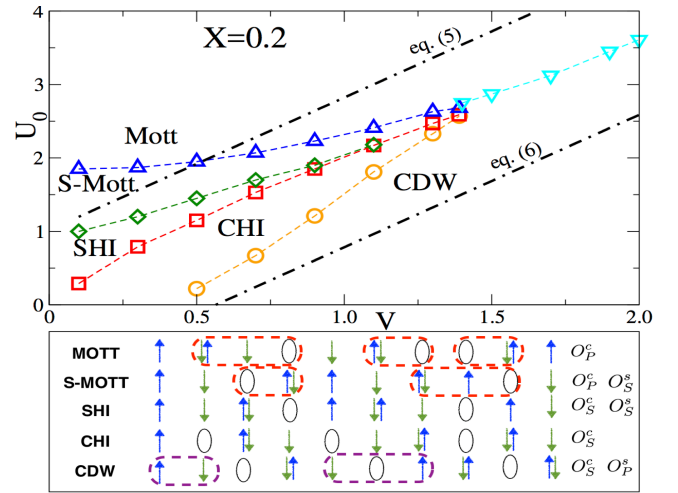


FIG. 1: (Color online) *Upper panel*: DMRG (symbols) and bosonization (dot-dashed line) phase diagram of (2) as a function of U_0 and V with $J = 1$ and $X = 0.2$. *Lower panel*: Cartoon of the phases with the relative NLOPs. The red (purple) dashed circles show the doublons-holons (spin up-down) virtual excitations

appears. When the effective model is treated within bosonization approach [32], its behavior appears to be well described in the charge sector; and only partly captured in the spin sector. Indeed, once quasi-exact density matrix renormalization group (DMRG) [34] calculations are performed, a further spin gapped region is found. Its presence can be ascribed to the coupling in this region of charge and spin degrees of freedom (DOFs). The diverse quantum phases are shown to be characterized by hidden magnetic order in the charge and, noticeably, in the spin DOFs. Crucially this magnetism can be solely detected by the non-vanishing of string-like nonlocal order parameters (NLOPs). Finally we discuss how all our achievements can be experimentally reproduced with the ongoing experimental setups involving magnetic atoms.

Model. We consider a balanced unit density two components (\uparrow and \downarrow) dipolar Fermi mixture of N particles with onsite periodically modulated interaction trapped in a 1D OL. Within a single band approximation, i. e. for their deep OL, the extended Fermi-Hubbard model (EFHM) [26] gives an accurate description of the system

$$H = -J \sum_{\langle ij \rangle} \sum_{\sigma=\uparrow,\downarrow} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + U(t) \sum_j n_{j\uparrow} n_{j\downarrow} + V \sum_{j,r \geq 1} \frac{n_j n_{j+r}}{r^3}, \quad (1)$$

where $\langle . \rangle$ denotes nearest neighbor (NN), $c_{j\sigma}$ ($c_{j\sigma}^\dagger$) destroys (creates) a σ -fermion in the j -th site of a lattice of length L and n_j ($n_{j,\sigma}$) counts the total (σ) number of particles. Crucially in cold atomic experiments all the couplings, namely the hopping rate J , the onsite interaction U and the long range dipolar repulsion V may be independently controlled by modifying the lattice depth, the transversal confinement [35, 36], using Feshbach resonances, and/or controlling the orientation and strength of the polarizing field. The time dependence in (1) can be induced by a rapid variation of the scattering length [37] producing a periodic modulation of the form $U(t) = U_0 + U_1 \cos(\omega t)$ which consequently makes $H(t) = H(t+T)$ being $T = 2\pi/\omega$. In the regime $\omega \gg U_0/\hbar, J/\hbar$, Floquet theory can be used [38] to approximately remove the time dependence. Indeed, analogously to the $V = 0$ case [39], we find that this kind of interaction modulation generates an effective time independent Hamiltonian where the hopping processes are renormalized by the density, namely the CHPs

$$H_{eff} = -J \sum_{\langle ij \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) \left(1 - X(n_{i\bar{\sigma}} - n_{j\bar{\sigma}})^2\right) + U_0 \sum_j n_{j\uparrow} n_{j\downarrow} + V \sum_{j,r > 0} \frac{n_j n_{j+r}}{r^3} \quad (2)$$

with $X = 1 - \mathcal{J}_0(U_1/\hbar\omega)$, \mathcal{J}_0 being the first kind Bessel function. The model (2) in the $V = 0$ regime has attracted huge interest in the contest of cuprate superconductors [40] while the $X = 0$ case, revealing the presence of spin Peirls dimerization (BOW), has been studied in the contest of time independent dipolar fermions, see [42] and references therein. Whereas only few partial analysis have tried to approach the full H_{eff} [43, 44], in case of just NN repulsion.

Nonlocal Order Parameters. In the contest of lattice fermions a very fundamental role is played by NLOPs of parity- and string-like form. Their correlation functions can be written respectively as

$$O_P^\nu(r) = \langle e^{i\pi \sum_{j < r} S_j^\nu} \rangle \quad (3)$$

$$O_S^\nu(r) = \langle S_l^\nu e^{i\pi \sum_{l \leq j < r} S_{l+r}^\nu} \rangle, \quad (4)$$

where $\nu = c, s$ refers to the charge and spin DOFs, and the charge and spin operators are defined as: $S_j^c = (1 - n_j)$ and $S_j^s = (n_{j\uparrow} - n_{j\downarrow})$. The relevance of NLOPs lies in the fact that they act as order parameters for gapped 1D phases[45] without breaking any continuous symmetry, thus in agreement with the Mermin-Wagner theorem [46]. Moreover, a non-vanishing string parameter is a signature of a phase with non-trivial topological order, which becomes unstable only in presence of a spontaneously broken symmetry [47]. These facts have motivated their intensive use to study 1D fermionic systems [31, 45, 48–50] helping to display physical properties not captured by the usual two-points correlation functions. More precisely in fermionic systems the role of the charge (spin) parity (3) is to signal the presence of Mott- (BEC-) like orders with correlated holon-doublon (spin up-down) virtual excitations [45] while the charge (spin) string (4) has the capability to capture hidden "dilute" holon-doublon (spin up-down) Haldane-like antiferromagnetic orders [1, 31].

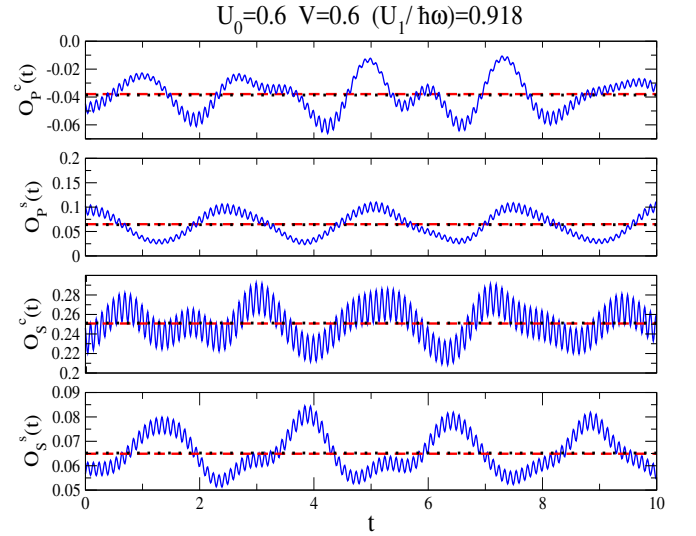


FIG. 2: (Color online) Blue continuous lines are the time evolution of NLOPs, red dashed lines are the time-averaged values of NLOPs and black dotted lines are the NLOP values given by H_{eff} eq. (2). All the results refer to a system of $L = 8$ sites and both couplings and inverse time are expressed in unit of J .

Dynamics vs Effective Model. In order to check the validity of the Floquet theory we compare the finite size NLOP values obtained by simulating both the time dependent (1) and the derived time independent effective model (2). The time-dependent simulations are performed by means of Exact Diagonalization starting from the initial ground state with couplings J, U_0, V and at $t > 0$ a time periodicity $U(t)$ is applied in order to get CHPs of strength $X = 0.2$. After that we monitor the time evolution and we evaluate the time-averages of the NLOPs [51]. As clearly shown in Fig. 2 all the time

averaged NLOP values are in good agreement with the ones obtained by exactly diagonalizing (2) with couplings J, U_0, V, X [52]. The latter result allows us to safely study H_{eff} in order to get the full phase diagram of (1) without losing some fundamental aspect encoded in the time dependence.

Luttinger analysis. As a first step we perform a bosonization analysis [32, 33] of H_{eff} . One passes to the continuum limit ($x = ja$, $a \rightarrow 0$ lattice constant) by replacing $c_{j\sigma} \rightarrow \sqrt{a}[(i)^j \psi_{R\sigma}(x) + (-i)^j \psi_{L\sigma}(x)]$, where $\psi_{A\sigma}(x) = \frac{\eta_{A\sigma}}{\sqrt{2\pi a}} e^{i\sqrt{\pi}[A\Phi_\sigma(x) + \Theta_\sigma(x)]}$, ($A = -, +$ for left and right movers respectively), α being an ultraviolet cutoff of the order of a . Here $\Phi_\sigma(x), \Theta_\sigma(x)$ are bosonic fields while $\eta_{A\sigma}$ are the fermionic Majorana Klein factors. In the weak coupling case and introducing the spin and charge fields $\phi_c(x) = \frac{1}{\sqrt{2}}(\Phi_\uparrow + \Phi_\downarrow)$, $\phi_s(x) = \frac{1}{\sqrt{2}}(\Phi_\uparrow - \Phi_\downarrow)$, together with their dual fields $\theta_\nu(x)$ ($\nu = c, s$), the Hamiltonian can be rewritten as the sum of three contributions $\mathcal{H} = \mathcal{H}_c + \mathcal{H}_s + \mathcal{H}_{cs}$. Each of the first two contributions has the form of a sine-Gordon model in the ν sector, namely $\mathcal{H}_\nu = \int dx [H_{0\nu} + \frac{m_\nu v_\nu}{2\pi a^2} \cos(\sqrt{8\pi}\phi_\nu)]$, with $H_{0\nu} = \frac{v_\nu}{2} [K_\nu (\partial_x \theta_\nu)^2 + \frac{1}{K_\nu} (\partial_x \phi_\nu)^2]$. Here, at unit filling and to first order in the interaction parameters, the mass term coefficients read $m_c = \frac{1}{2\pi} \left(-U + \frac{3\zeta(3)V}{2} + 16\frac{X}{\pi} \right)$, $m_s = \frac{1}{2\pi} \left(U - \frac{3\zeta(3)V}{2} + 16\frac{X}{\pi} \right)$, whereas the Luttinger parameters are $K_c = 1 - \frac{1}{4\pi} [U + \frac{11}{2}\zeta(3)V - \frac{16X}{\pi}]$, $K_s = 1 + \frac{m_s}{2}$; moreover $v_\nu = v_{0F} [2 - \frac{X}{2} - K_\nu]$. The last contribution reads $\mathcal{H}_{cs} = \frac{M_{cs} v_{0F}}{2\pi a^2} \int dx \cos(2\sqrt{2\pi}\phi_c) \cos(2\sqrt{2\pi}\phi_s)$, with $M_{cs} = \frac{1}{2\pi} [\frac{3}{2}\zeta(3)V - \frac{8X}{\pi}]$; it couples the spin and the charge sector. In a renormalization group (RG) analysis the term is highly irrelevant in the weak-coupling limit, having scaling dimension 4. It can safely be neglected in deriving the opening of gapped phases in the Luttinger liquid regime, whereas in presence of a gapped sector it renormalizes the mass term of the gapless sector shifting the transition line to the fully gapped phase [43]. Notice that for $V = \frac{16}{3\pi\zeta(3)}X$ the coupling of the charge and spin DOFs becomes identically vanishing. In case the spin-charge coupling is dealt as above, the massive phases can be analyzed in the asymptotic limit by studying the RG flow equations of the two decoupled sine Gordon models. In each sector, the transition line is identified by the competition of the kinetic and the mass terms: explicitly the ν -sector is gapless only when $2(K_\nu - 1) \geq |m_\nu|$. The obtained weak coupling phase diagram is reported in Fig. 1 (dot-dashed lines). For non-vanishing V , a charge gapped phase is always present, except at a critical values U_c ,

$$U_c = \frac{3}{2}\zeta(3)V + \frac{16}{\pi}X, \quad (5)$$

at which the phase changes from a Mott insulating phase with $O_P^c \neq 0$ to a charge Haldane insulator (CHI) with $O_S^c \neq 0$. Moreover, a spin gapped phase opens only

for U values lower than

$$U_s = \frac{3}{2}\zeta(3)V - \frac{16}{\pi}X, \quad (6)$$

implying that, for any non-vanishing $X > 0$, in the range $U_s < U < U_c$ the ground state consists of a partly gapped phase characterized solely by charge hidden magnetism, as suggested in [31]. The results obtained above can be integrated in the intermediate coupling regime by incorporating part of the interaction in an exact way [42]. Such study predicts for instance the closure of the CHI at the critical value V_c reported in figure.

Summarizing, we find that bosonization predicts the presence of three insulating phases. One of them has finite O_P^c , whereas the other two are characterized by $O_S^c \neq 0$. Of the two latter, one has additional spin gap signaled by a finite O_P^s , thus configuring as a charge density wave (CDW) ordered phase [31]. The other remains partly gapped, thus configuring as a CHI with hidden magnetic charge order up to V_c .

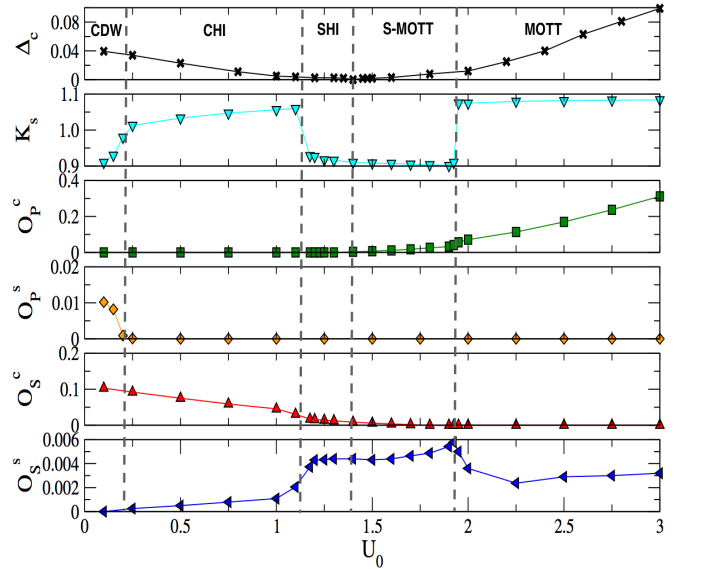


FIG. 3: (Color online) TDL of charge gap, spin Luttinger constant and NLOPs of (2) for $J = 1$, $V = 0.5$ and $X = 0.2$ as a function of U_0 . Δ_c is extrapolated by using open boundary conditions and sizes up to $L = 44$. K_s and the NLOPs are extrapolated by using periodic boundary conditions and sizes up to $L = 36$. In all our DMRG simulations we cut r to three NN keeping up to 1600 DMRG states and performing up to 6 finite size sweeps.

DMRG results. The bosonization analysis is expected to give reliable results in the weak to intermediate coupling regime. Below we perform a further analysis based on quasi exact DMRG simulations to explore the full phase diagram. As a first step we evaluate the thermodynamic limit (TDL) of the charge gap $\Delta_c = (E(N+2) + E(N-2) - 2E(N))/2$ being $E(N)$ the ground state energy of N particles. Fig. 3 clearly shows that Δ_c vanishes only in the point where the string-like charge order

is substituted by parity-like charge order thus signaling a continuous phase transition. DMRG results become more crucial when analyzing the behavior in the spin DOFs, where CHPs are known to make bosonization less predictive [41]. As a first step we evaluate the TDL of the Luttinger spin constant $K_s = \lim_{q \rightarrow 0} \pi S^s(q)/q$ where $S^s(q) = 1/L \sum_{k,l} e^{iq(k-l)} (\langle S_k^s S_l^s \rangle - \langle S_k^s \rangle \langle S_l^s \rangle)$ is the spin structure factor. Luttinger theory predicts $K_s = 1(0)$ in absence (presence) of a spin gapped phase. Here both logarithmic corrections and finite size effects make very difficult to get sharp 0,1 values. Nevertheless a well established and accurate approximation, see [53], is to consider a spin gapless (gapped) phase in presence of $K_s > 1(< 1)$: the transition point is then fixed by the crossing of the value 1. As clearly visible in Fig. 3 the analysis based on the TDL of K_s surprisingly finds, for small values of V , a further large spin gapped phase ranging in a region around the point where $\Delta_c = 0$. The result allows us to better characterize part of the phase diagram. Indeed as expected for large V a phase with CDW order is found in analogy with the NN EFH model (see for instance [43]). The similarities extend also to the large U_0 region where a charge gapped Mott phase signaled by O_P^c is present. Between the CDW and Mott regions instead, the fully gapped BOW phase predicted by the EHF model and signaled by both parity operators is destroyed by the CHPs. Indeed Fig 3 shows that, at intermediate U_0, V , three different phases characterized by hidden magnetism take place. In particular as predicted by bosonization, a phase having as an order parameter the only O_S^z appears (CHI). The latter reproduces in a two-species fermionic system the same charge hidden antiferromagnetic order of the well known topological Haldane phase studied in the context of spin-1 chains [1], extended Bose-Hubbard model [54–56], and multicomponent fermions [57]. The main point now is to describe in terms of spin NLOPs the two remaining fully gapped regimes having different charge orders. Fig. 3 clearly shows that these two phases are properly characterized by a non-vanishing O_S^z , meaning that spin \uparrow and \downarrow fermions are alternated and diluted in an arbitrary number of holons and doublons. In particular, at fixed V and by increasing U_0 , we first find a phase, called spin Haldane insulator (SHI), with the two strings being simultaneously non-vanishing, thus describing hidden magnetism in both DOFs. At the same time, by a further increasing the onsite interaction, the hidden antiferromagnetic charge order is destroyed by the Mott-like charge order. Whereas O_S^z remains finite in a further range, thus giving rise to spin gapped Mott phase (S-Mott). We verified that the three phases CHI, SHI and S-Mott with hidden magnetism are present for different values of X . This shows that CHPs act as the responsible for the formation of these exotic antiferromagnetic orders which, at our best knowledge, have never been predicted. The above explanation relies crucially on the role

of the NLOPs since two points correlation functions are not able to capture hidden orders and thus to properly describe what happens in the considered system (2).

Experimental realization. The previous quantum phases could be studied by using a mixture of Erbium isotopes. In particular fermionic ^{167}Er [58] as well as bosonic ^{168}Er [27] isotopes are currently available in laboratories. The scattering length of the ^{168}Er can be accurately tuned to reach an effective hard-core regime, thus giving rise to a two components Fermi mixture. At fixed $\theta = 0$ (θ being the angle between the orientation of the dipoles and the interparticles distance,) a recoil energy $E_R = h \times 4.3\text{KHz}$, and an appropriate lattice depth should allow to easily achieve the value $V/J \sim 1$ which is exactly the regime where hidden magnetism is predicted. Feshbach resonance to tune ^{167}Er - ^{168}Er onsite interaction should become available [59] and, in order to get correlated hopping processes, a rapid time dependent modulation can be applied following the procedure in [17]. Noticeably charge and spin gaps can be in principle detected by means of lattice modulation and RF spectroscopy respectively. Moreover parity charge operator has been already measured by means of in-situ imaging [60]. The combination of three latter ingredients, namely O_P^c and charge and spin gaps, are sufficient to probe at least our phase boundaries. Measure of O_S^z would require resolving doublons which could be in principle done following the proposal in [13]. Spin NLOPs instead have not been measured yet, though the very recent site-resolved observations of antiferromagnetic correlations [61] could open the path towards their detection.

Summary. We have shown that periodical onsite modulation of lattice dipolar fermions allows to realize non trivial Hamiltonians with long range dipolar interaction and correlated hopping processes terms. The latter drive the systems from a static BOW configuration to equilibrium phases with equilibrated hidden magnetism, which could appear in the charge, spin or both sectors. These phases can be detected with the currently available experimental setups and probes, and are expected to exhibit non-trivial topological features.

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